2.10 Slip Line Method (SLM)(すべり線解析)
Stress Characteristic Method（特性曲線法）

For 2 D (plane strain) problem

two stress equilibrium equations (4.48-4.51)

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

○ three unknown values

three equations

failure criteria (Mohr-Coulomb)

$\sigma_1 - \sigma_3 = 2c_u$ for undrained conditions (ϕ_u=0)

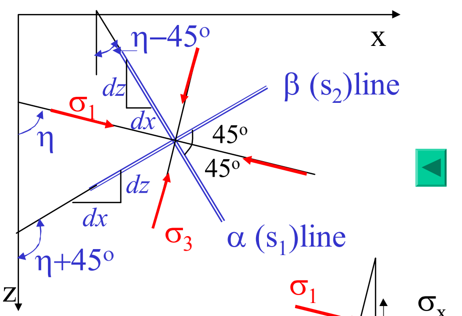
$\sigma'_1 - \sigma'_3 = 2c'\cos\phi' + (\sigma'_1 + \sigma'_3)\sin\phi'$ for drained conditions

Chapter 7

hyperbolic type simultaneous partial equations in s and η with respective to slip lines (α and β slip lines)

+ **boundary conditions** → **solutions** ***accurate but not easy to apply***

Directions of slip plane in **undrained** loading in 2D

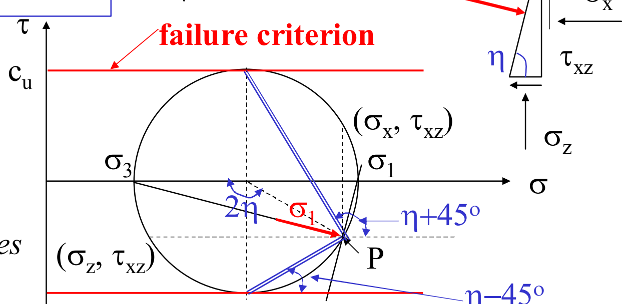


Geometrical condition of slip lines

$$\alpha(s_1)line: \frac{dx}{dz} = \tan(\eta - 45^\circ)$$
$$\beta(s_2)line: \frac{dx}{dz} = \tan(\eta + 45^\circ) \quad (42)$$

a pair of slip lines is always orthogonal

possible shape:
ex)
orthogonal straight lines
circular arc=fan



Stress field for undrained loading

unknown values

$\sigma_x, \sigma_z, \tau_{xz} \longrightarrow \sigma_1 - \sigma_3 = 2c_u$ (43) where $s = \frac{1}{2}(\sigma_x + \sigma_z)$

failure condition

$$\sigma_x = s - c_u \cos 2\eta$$
$$\sigma_z = s + c_u \cos 2\eta \qquad (44)$$
$$\tau_{xz} = c_u \sin 2\eta$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma$$
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad (45)$$

Substituting eqs.(44) into eqs.(45)=> a pair of simultaneous partial differential equations in s and η.

Hyperbolic type equation: => characteristics line= slip line

$$\frac{\partial s}{\partial z} - 2c_u \sin 2\eta \frac{\partial \eta}{\partial z} + 2c_u \cos 2\eta \frac{\partial \eta}{\partial x} = \gamma$$
$$\frac{\partial s}{\partial x} + 2c_u \cos 2\eta \frac{\partial \eta}{\partial z} + 2c_u \sin 2\eta \frac{\partial \eta}{\partial x} = 0 \qquad (46)$$

not easy to treat, new coordinate system *characteristics* $(x,z) \Rightarrow (s_1, s_2)$

Directional derivative

derivation of composite function: $s_1 = f(x,z), s_2 = f(x,z)$

$$\frac{\partial}{\partial s_1} = \frac{\partial}{\partial x}\frac{\partial x}{\partial s_1} + \frac{\partial}{\partial z}\frac{\partial z}{\partial s_1} = \frac{\partial}{\partial x}\sin(\eta - 45^\circ) + \frac{\partial}{\partial z}\cos(\eta - 45^\circ)$$
$$\frac{\partial}{\partial s_2} = \frac{\partial}{\partial x}\frac{\partial x}{\partial s_2} + \frac{\partial}{\partial z}\frac{\partial z}{\partial s_2} = \frac{\partial}{\partial x}\sin(\eta + 45^\circ) + \frac{\partial}{\partial z}\cos(\eta + 45^\circ) \qquad (47)$$

from eqs.(47)

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial s_1}\sin(\eta - 45^\circ) + \frac{\partial}{\partial s_2}\cos(\eta - 45^\circ)$$
$$\frac{\partial}{\partial z} = \frac{\partial}{\partial s_1}\cos(\eta - 45^\circ) - \frac{\partial}{\partial s_2}\sin(\eta - 45^\circ) \qquad (48)$$

stress condition of slip lines: ***KGtter's*** equation on ϕ_u=0 material

eq.(48) =>
eq.(46)

$$\left(\frac{\partial s}{\partial s_1} - 2c_u \frac{\partial \eta}{\partial s_1}\right)\cos(\eta - 45^\circ) - \left(\frac{\partial s}{\partial s_2} + 2c_u \frac{\partial \eta}{\partial s_2}\right)\sin(\eta - 45^\circ) = \gamma$$
$$\left(\frac{\partial s}{\partial s_1} - 2c_u \frac{\partial \eta}{\partial s_1}\right)\sin(\eta - 45^\circ) + \left(\frac{\partial s}{\partial s_2} + 2c_u \frac{\partial \eta}{\partial s_2}\right)\cos(\eta - 45^\circ) = 0 \qquad (49)$$

from (49)

$$\frac{\partial s}{\partial s_1} - 2c_u \frac{\partial \eta}{\partial s_1} = \gamma \cos(\eta - 45^\circ)$$

$$\frac{\partial s}{\partial s_2} + 2c_u \frac{\partial \eta}{\partial s_2} = \gamma \sin(\eta - 45^\circ)$$

(50) ⟺ ordinary differential eqs.

finite difference form of eq.(50)

$$\Delta s = 2c_u \Delta\eta + \gamma\Delta s_1 \cos(\eta - 45^\circ)$$

$$\Delta s = -2c_u \Delta\eta + \gamma\Delta s_2 \sin(\eta - 45^\circ) \quad (51)$$

from $\Delta s_1 \cos(\eta - 45^\circ) = \Delta s_2 \sin(\eta - 45^\circ) = \Delta z$

$\Delta s = 2c_u \Delta\eta + \gamma\Delta z$:α (s_1) slip line

$\Delta s = -2c_u \Delta\eta + \gamma\Delta z$:β (s_2) slip line

(52) = (7.19&40)

Change of mean stress in a slip line is given by a linear combination of **rotation of σ_1 ($\Delta\eta$),** i.e., change of the inclination of slip line, and **change of elevation (Δz).**

construction of slip line network => stress condition in the ground

Solutions of slip line method for ϕ_u=0 material

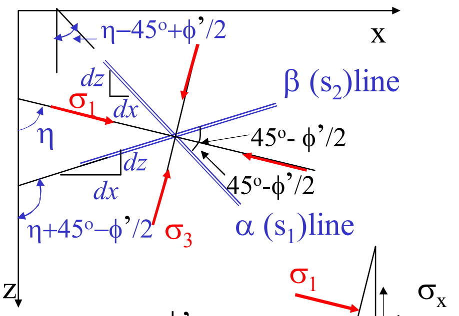
Construction of slip line network with given boundary condition:

For ϕ_u=0 material with constant c_u, the slip line can be made relatively easier. (see text. 7.8, p264-269) But for non-uniform strength condition and complicated boundary conditions, numerical approach using finite difference approximation is often used to construct slip line network.

Home work 5 : **due date 15th of Nov.**

Obtain active earth pressure on vertical rough retaining wall. (see Chap. 7.8 3. p267-269)

Directions of slip plane in **drained loading** in 2D

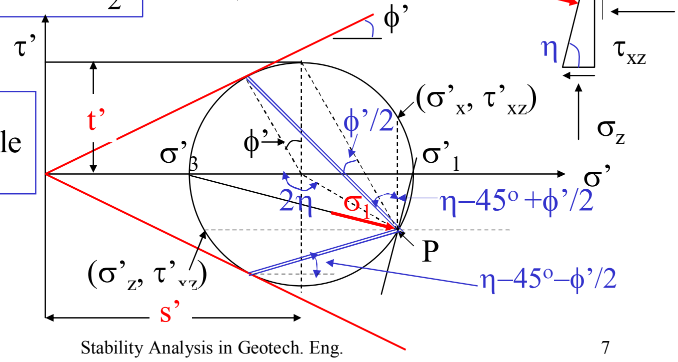


Geometrical condition of slip lines

$$\alpha(s_1)line: \quad \frac{dx}{dz} = \tan(\eta - 45^\circ + \frac{\phi'}{2})$$

$$\beta(s_2)line: \quad \frac{dx}{dz} = \tan(\eta + 45^\circ - \frac{\phi'}{2}) \qquad (53)$$

a pair of slip lines intersect by the angle of 90°-ϕ'



possible shape:

ex) *straight lines*

log-spiral =fan

Stress field for drained loading

$$s' = \frac{1}{2}(\sigma'_x + \sigma'_z), t' = \frac{1}{2}(\sigma'_1 - \sigma'_3)$$

unknown values

σ'_x, σ'_z, τ'_{xz} ⟶ $t' = s' \sin\phi'$ (54) ⟶

failure condition

$$\sigma'_x = s'(1 - \sin\phi' \cos 2\eta)$$
$$\sigma'_z = s'(1 + \sin\phi' \cos 2\eta) \qquad (55)$$
$$\tau'_{xz} = s' \sin\phi' \sin 2\eta$$

$$\frac{\partial \sigma'_z}{\partial z} + \frac{\partial \tau'_{xz}}{\partial x} = \gamma - \frac{\partial u}{\partial z}$$
$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau'_{zx}}{\partial z} = 0 \qquad (56)$$

Substituting eqs.(55) into eqs.(56)=> a pair of simultaneous partial differential equations in s and η.

Stress field for drained loading

$$
\begin{aligned}
&(1+\sin\phi'\cos 2\eta)\frac{\partial s'}{\partial z}+\sin\phi'\sin 2\eta\frac{\partial s'}{\partial x}\\
&-2s'\sin\phi'\sin 2\eta\frac{\partial \eta}{\partial z}+2s'\sin\phi'\cos 2\eta\frac{\partial \eta}{\partial x}=\gamma-\frac{\partial u}{\partial z}\\
&\sin\phi'\sin 2\eta\frac{\partial s'}{\partial z}+(1-\sin\phi'\cos 2\eta)\frac{\partial s'}{\partial x}\\
&+2s'\sin\phi'\cos 2\eta\frac{\partial \eta}{\partial z}+2s'\sin\phi'\sin 2\eta\frac{\partial \eta}{\partial x}=-\frac{\partial u}{\partial x}
\end{aligned}
\qquad (57)
$$

with the same technique as used for undrained loading

Hyperbolic type equation: => characteristics line= slip line

$$
\begin{aligned}
\Delta s' &= 2s'\tan\phi'\Delta\eta+\gamma(\Delta z+\tan\phi'\Delta x) && :\alpha\ (s_1)\text{ slip line}\\
\Delta s' &= -2s'\tan\phi'\Delta\eta+\gamma(\Delta z-\tan\phi'\Delta x) && :\beta\ (s_2)\text{ slip line}
\end{aligned}
\qquad (58)
$$

Change of mean stress in slip lines for drained loading

$$
\begin{aligned}
\Delta s' &= 2s'\tan\phi'\Delta\eta+\gamma'(\Delta z+\tan\phi'\Delta x) && :\alpha\ (s_1)\text{ slip line}\\
\Delta s' &= -2s'\tan\phi'\Delta\eta+\gamma'(\Delta z-\tan\phi'\Delta x) && :\beta\ (s_2)\text{ slip line}
\end{aligned}
\qquad (58)
$$

Change of mean stress in a slip line

changing exponentially

= product of mean stress (s) x rotation of σ_1 ($\Delta\eta$)

+

change of elevation (Δz) and change of horizontal location(Δx)

more complicated than undrained loading

$$
\begin{aligned}
\Delta s &= 2c_u\Delta\eta+\gamma\Delta z && :\alpha\ (s_1)\text{ slip line}\\
\Delta s &= -2c_u\Delta\eta+\gamma\Delta z && :\beta\ (s_2)\text{ slip line}
\end{aligned}
\qquad \textbf{(52)}
$$

For slip line network construction, sketching method cannot be applied except of very simple conditions.

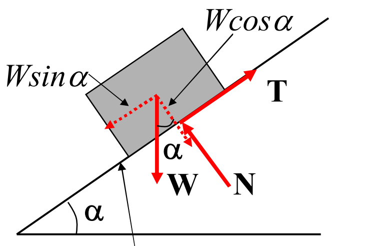
e.g. straight lines, γ=0

2.11 Limit Equilibrium Method (LEM)

（極限平衡法）

Basics of LEM=Statics of rigid body

rigid body on a straight slope



μ_{cr}:critical friction coefficient

From equilibrium in the directions parallel and normal to the slope,

$$N = W\cos\alpha$$
$$W\sin\alpha = T = \mu N \quad \textbf{(59)}$$

force to drive moving caused by gravity ($W\sin\alpha$)

force to resist moving mobilized by friction on the surface (μN)

mobilized friction coefficient: $\mu = \dfrac{\mu_{cr}}{F_s}$ **(60)**

(60)=>(59)

$$W\sin\alpha = \mu W\cos\alpha = \frac{\mu_{cr}}{F_s} W\cos\alpha \quad \textbf{(61)}$$

$$F_s = \frac{\mu_{cr} W\cos\alpha}{W\sin\alpha} \quad \textbf{(62)}$$

maximum resisting force against sliding ($\mu_{cr} W\cos\alpha$)

force to drive sliding ($W\sin\alpha$)

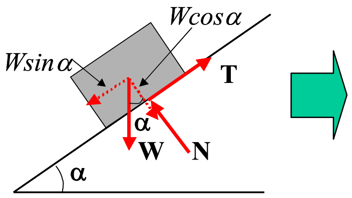
Eq.(62) is equivalent to the factor of safety on shear strength used in slope stability analysis.

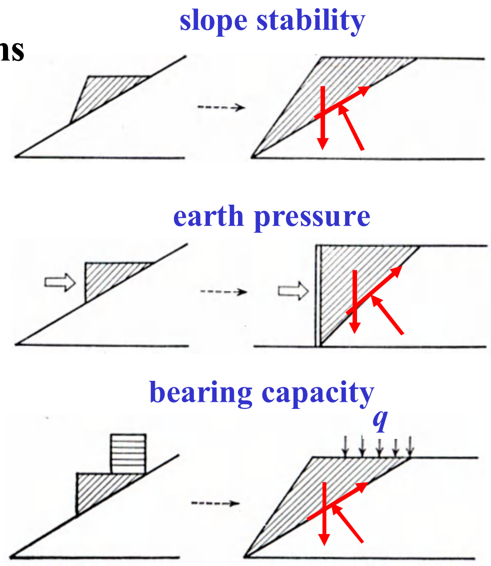
$$F_s = \frac{\tau_f}{\tau_m} \quad \textbf{(63)}$$

shear strength (τ_f)

mobilized shear stress (τ_m)

Statics of rigid body
vs.
Classical stability problems





LEM can be applied for various stability problems of geotechnical structure.

Procedures of Limit Equilibrium Method

1. Failure mechanism
-assuming rigid body
-moving along a constraint surface:*straight, circular, log-spiral*

similar to upper bound analysis
but circular surface for ϕ>0

2. Force-equilibrium with respective to the rigid body at critical condition (*failure criteria*)

3.Optimization
minimization, maximization

similar to lower bound analysis
but only equilib. of rigid body not in side and outside of the body

no restriction on failure mechanism (incompatible mechanism: OK)
not theoretically rigorous but practically most applicable

ex: Coulomb's earth pressure theory
Slice methods

LEM and UBA give the same answers with the same mechanism, if the forces acting to the rigid bodies are properly evaluated in LEM.

ex). Earth pressures:

P_p for ϕ=0, (5.45) on p.161 and (8.8) on p.289

P_a for ϕ>0, (6.30) on p.203 and (8.16) on p.291

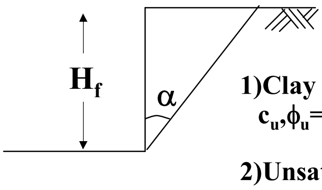
P_p for ϕ>0, (6.40) on p.206 and (8.17) on p.291

Bearing capacity

for ϕ=0, (5.57) on p.165 and (8.22) on p.298

Home work 6 (due date:15th of Dec.)

(1) Obtain the failure height (H_f) of vertical slope of 1)clay with c_u , ϕ_u=0 and γ_{sat} and 2)unsaturated soil with c, ϕ and γ_t by using LEM and UBA with straight slip line.



1)Clay with c_u,ϕ_u=0 and γ_{sat}

2)Unsaturated soil with c, ϕ and γ_t